Composites Science and Technology 69 (2009) 1790–1796

Contents lists available at ScienceDirect

Composites Science and Technology

journal homepage: www.elsevier.com/locate/compscitech

Structural analysis of small span textile reinforced concrete shells with double curvature

Tine Tysmans *, Sigrid Adriaenssens, Heidi Cuypers, Jan Wastiels

Vrije Universiteit Brussel, Faculty of Engineering Sciences, Department of Mechanics of Materials and Constructions (MeMC), Pleinlaan 2, 1050 Brussels, Belgium

ARTICLE INFO

Article history:
Received 6 January 2008
Received in revised form 2 September 2008
Accepted 15 September 2008
Available online 21 September 2008

Keywords:
A. Ceramic–matrix Composites
A. Textile composites
C. Finite element analysis
A. Glass fibres

ABSTRACT

The reinforcement of cement with glass fibre textiles imbues the composite with a tensile as well as compressive load-bearing capacity. The tensile capacity allows the elimination of steel reinforcement as well as the concrete corrosion cover in structural applications. With textile reinforced concrete (TRC) thin and/or free form shells could be realized. In this paper, a parametric study is used to evaluate the structural applicability of TRC for small span (2–15 m) doubly curved roof shells. The application of two different, existing TRC material combinations demonstrates the influence of the applied composite material on the design of the shell.

1. Introduction

With the advent of geometric CAD modelling tools, the design of distinctive buildings nowadays tends towards creative, eye-catching free forms. In this context, slenderness and resulting weight reduction in large span covers is a major design consideration. The increased use of lightweight membrane structures, allowing for creative forms, demonstrates this evolution. However, the use of membranes has its disadvantages: firstly, the necessary membrane pre-stress requires strong and expensive anchorage. Secondly, the use of textiles leads to problems of thermal and acoustical insulation and fire-safety.

Steel-reinforced concrete shells do not show these problems. Slender large span shells result from designs that aim at a uniform stress state and limit bending action. Shells moreover owe their structural stiffness to their curvature (geometric stiffness) rather than to their thickness. The form freedom of steel-reinforced concrete shells is however limited. Concrete has relatively high compression strength, but a very low tensile capacity. Up to now, concrete shell design is therefore restricted to forms working mainly under compressive stresses. Tensile reinforcement is still required, since tensile stress zones can still occur under certain load combinations. The practical difficulties of positioning and shaping traditional steel reinforcement restrict the form freedom. Moreover, the high cost associated with placing the steel reinforcement and the formwork, makes freeform reinforced concrete shells an uneconomical solution. Another disadvantage of using steel reinforcement is the necessity of a concrete corrosion cover. The minimum corrosion cover (depending on the exposure coefficient [1]) results in an increased dead weight and limits the minimum shell thickness (to approximately seventy millimetres). Therefore, shell forms are rarely used to cover small spans. The replacement of the steel bars by a flexible, non-corrosive reinforcement can eliminate the disadvantages of steel reinforcement and permit thin, free form shells. Reinforcement flexibility can be obtained when the reinforcement diameter is considerably reduced and at the same time the volume fraction of the reinforcement is retained. The use of fibres is a possible solution to fulfil these requirements.

The addition of short glass fibres to a concrete ("premix method") constrains the crack width and enhances the toughness of the initially brittle material. However, with this concrete production technology, only a limited amount of fibres can be inserted, and thus the resulting tensile capacity is limited. Moreover, the orientation and spatial distribution of the fibres are usually not well controlled [2]. According to classical composite modelling methods, high amounts of fibres have to be present in the concrete matrix in the direction of the tensile stresses to provide the necessary post-cracking stiffness and strength [3]. These high fibre volume fractions are in this study achieved by adding continuous randomly oriented glass fibres in the form of textiles to a fine tuned concrete matrix by typical composite production technologies (hand lay-up, pultrusion). Earlier research has demonstrated the structural potential of glass fibre textile reinforced concrete (TRC) in thin building elements, such as sandwich panels [4], modular hyperbolic paraboloid sandwich panels [5] and large-size façade elements [6].

This paper focuses on the structural potential of randomly oriented glass fibre textile reinforced concrete in small span doubly

* Corresponding author. Tel.: +32 2 6292921; fax: +32 2 6292928.
E-mail address: ttysmans@vub.ac.be (T. Tysmans).

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doi:10.1016/j.compscitech.2008.09.021

curved shells. Specifically this paper examines whether when using textiles exclusively as reinforcement in fine grained concrete, the tensile capacity of the composite is sufficiently high to realise small span shells, which are thinner and thus lighter than when constructed with conventional steel-reinforced concrete. Therefore, a doubly curved shell shown in Fig. 1, is designed with a force-density optimisation of a cable net (equal force-density in all cables of the net), which allows the shell to be cast on a flexible membrane formwork. Initially, the structural behaviour of a two meter span doubly curved shell is analysed with the Finite Element Method (FEM) under realistic load combinations of self weight, wind and snow. This analysis demonstrates the importance of the cementitious composite's tensile capacity for its application in small span doubly curved shells. The smallest span (2 m) was chosen since it can easily be manufactured in a laboratory. Several shells will be constructed in the near future to validate the obtained numerical with the experimental results. In this study, the span width of the doubly curved shell is gradually increased to more realistic sizes (10 and 15 m), until the shell thickness reaches the same order of magnitude as for thin steel-reinforced concrete shells. In this way, the span range is determined in which the use of TRC instead of traditional steel-reinforced concrete leads to thinner and lighter doubly curved shells. With the advent of TRC, free form shell structures can more easily be manufactured. This paper demonstrates however a second great advantage: shells in TRC can be made only as thin as structurally necessary, such that they show an efficient material use even for smaller spans (up to 15 m). Preliminary design rules based on the Euler-Bernoulli beam equation are formulated, which determine the influence of the span and the composite's tensile strength on the shell thickness. The influence of the material behaviour on the shell thickness is demonstrated by applying two different randomly oriented glass fibre textile – fine grained concrete combinations to the shell.

2. Glass textile reinforced concrete: material behaviour

Textile reinforced concrete has different mechanical behaviour in tension and compression. In compression, the composite can be considered to behave linearly elastic up to fracture. In tension however, cracks initiate and propagate in the brittle matrix composite at very low stress levels. Fig. 2 shows the nonlinear stress–strain behaviour in tension of TRC. Three stages can be distinguished in this curve [2,7]:

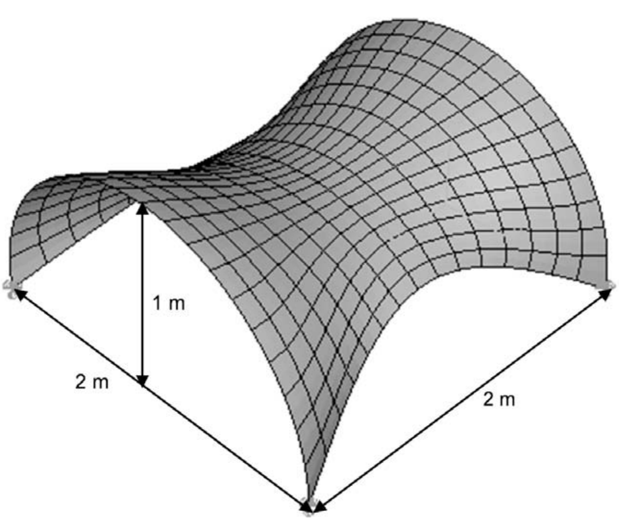

Fig. 1. Geometry and mesh of 2 m span doubly curved TRC shell.

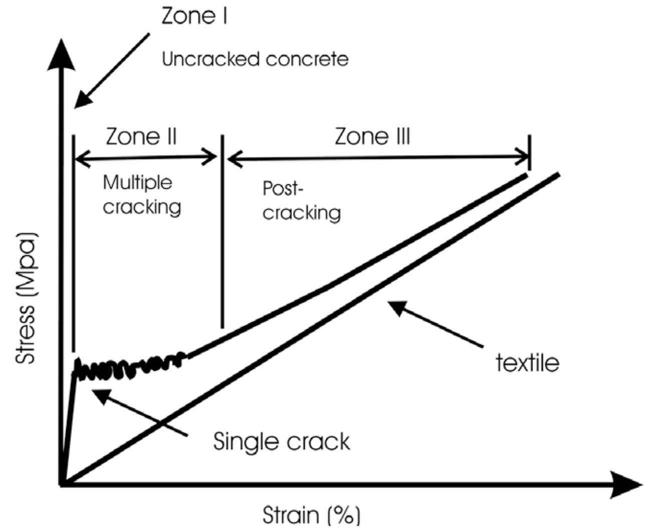

Fig. 2. Typical stress–strain diagram of textile reinforced concrete under uni-axial tensile loading.

- *Pre-cracking zone (zone I):* In this linear elastic zone, the "law of mixtures" for linear elastic composites can be used to determine the stiffness E_1 of the non-cracked composite. Since the volume fraction of the glass fibres is moderate (maximum 20%) and the stiffness of matrix and fibres are of the same order of magnitude, the stiffness in this first stage corresponds approximately to the E-modulus of the matrix.
- *Multiple-cracking zone (zone II):* When the tensile strength of the concrete matrix is exceeded (which occurs at approximately one percent of the tensile strength of the fibres), the first cracks appear. At the crack face, the tension force has to be carried by the reinforcement. As the tension force increases, additional cracks are initiated and propagated along the whole composite (multiple cracking). At the vicinity of a crack face, the fibres and matrix are debonded and the previously existing adhesive bond is replaced by a frictional bond. Due to this frictional stress transfer between fibres and concrete, forces are further transferred in the matrix until the concrete matrix' tensile strength is reached again and a neighbouring crack develops. The crack spacing and the crack width are determined by three factors namely the fibre reinforcement, the bond characteristics between reinforcement and concrete, and the concrete failure strain.
- *Post cracking zone (zone III):* A stabilised crack pattern state is created and no further cracks occur. When the load further increases, the fibres are strained further until the composite fails by fibre failure or fibre pull-out. The matrix stresses remain constant in this zone and only the fibres contribute to the composite's stiffness.

Since glass fibres are economical in comparison to other fibre materials with similar stiffness and strength properties, they are favoured when inserting large quantities into cement. Glass fibres have however one serious disadvantage: they are chemically attacked by the cement matrices' alkalinity. Generally, this particular problem is solved in two ways: firstly the use of alkali-resistant glass fibres, which do not eliminate but slow down the degradation process [8], and secondly the use of a low-alkaline or non-alkaline matrix. Both these randomly oriented glass fibre textile - concrete combinations are considered in this study: on the one hand, the alkali-resistant glass fibre textiles with an Ordinary Portland Cement fine tuned mortar (GTR-OPC, [9]) and on the other hand E-glass fibre textiles with a high viscosity cement which shows a neutral pH after hardening: Inorganic Phosphate Cement (GTR-IPC, [10]). The loss of strength due to ageing is proven to be considerably lower when using an IPC matrix than when using an OPC-matrix [11].

The studied randomly oriented glass fibre textile – fine grained concrete combinations have different mechanical properties. OPC fine tuned mortar consists of relatively large grains (order of magnitude of 1 mm), as a result of which only relatively small fibre volume fractions (5%) can be inserted. Inorganic Phosphate Cement (IPC) with its high viscosity and very fine grains (order of magnitude 10–100 μm), can have larger volume fractions (up to 20%) introduced. In plane randomly oriented glass fibre textiles are preferred, as the direction and magnitude of stresses in shells vary with the occurring load combination. Fig. 3 shows the stress-strain behaviour of GTR-OPC with 5 vol% of fibres and GTR-IPC with a fibre volume of 13%. Both materials are assumed to be isotropic throughout the composite due to the use of randomly oriented fibre textiles. The advantages and disadvantages of both material combinations can be concluded as follows: on the one hand, the IPC-matrix has a lower uncracked stiffness and tensile strength than OPC, meaning that the pre-cracking zone of GTR-IPC extends to lower stresses, and the initial composite stiffness E_1 is lower than for GTR-OPC. On the other hand, due to the high fibre volume fraction, a higher design composite tensile strength is obtained for GTR-IPC (20 MPa) than for GTR-OPC (10 MPa). This design strength was obtained by calculating the characteristic strength (5% percentile) based on experimental tests, followed by the application of a material safety factor. It must be emphasized that this experimentally determined averaging composite's tensile strength is considerably lower than the theoretical strength considering the full fibre strength. This difference in strength is among other things due to the bond properties of the composite and the variation of the fibre properties [7]. Material design properties (including safety factors) of both glass fibre textile – fine grained concrete combinations used in this study are summarised in Table 1.

3. Modelling of TRC small span doubly curved shell and calculation hypotheses

This paper carries out a parametric study to evaluate the structural potential of TRC for small span doubly curved shells. Three shells with varying dimensions – span 2 × 2 m (Fig. 1), 10 × 10 m and 15 × 15 m – but constant height/span ratio of 1/2 are analysed with FEM under realistic load combinations and structurally evaluated. The minimum thickness is determined for every shell to resist all load combinations and to limit the deformations to the maximum limit specified in Eurocode 2: Design of Concrete Structures [12]. For the 2 m span shell, the thickness is modified in a discrete way per 1 mm; for the 10 and 15 m span shells, the thickness is adapted per 5 mm.

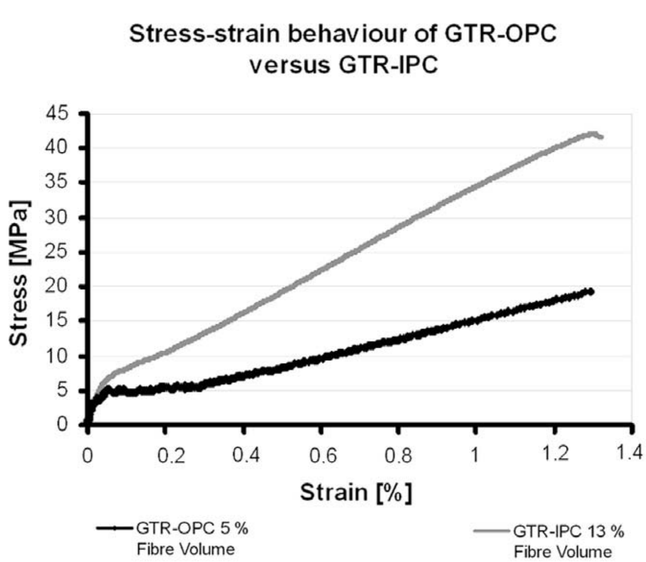

Fig. 3. Stress–strain behaviour of GTR-OPC versus GTR-IPC under uni-axial tensile loading.

Table 1
Material properties of GTR-IPC and GTR-OPC

			GTR-IPC	GTR-OPC
Fibre density	V_f	Volume%	13%	5%
Density	ρ	kg/m^3	1900	1900
Young's modulus I	E_1	GPa	18	30
Poisson coefficient	ν		0.3	0.3
Design tensile stress	$\sigma_{t,\ max}$	MPa	20	10
Design compressive stress	$\sigma_{c,\ max}$	MPa	25	25

Since this paper comprises a preliminary study of the possibilities of concrete composite shells, it focuses only on static analyses. Dynamic and fatigue analyses will be part of a further study. From previous studies it was found that repeated loading leads to a reduction of the material stiffness at all stress levels [13]. Consequently, the deflections might be larger than calculated through a pure static analysis. The influence of repeated loading on the findings in this paper will be discussed further in Section 4.1.

3.1. Doubly curved shell modelling

Fig. 1 illustrates the geometry of the 2 m span shell. All three shells are hinged at their four corners in all *x*, *y* and *z*-direction while the edges are free. These support conditions lead to stress concentrations in the four corners. Therefore when constructing the doubly curved shell the support surface will be enlarged to alleviate these concentrations. The global minimum thickness of the shells is thus determined without considering these stress concentrations. Tensile stresses are indicated as being positive by convention, compressive stresses as being negative.

The shells are modelled using the commercially available FEM software package Abaqus (version 6.5-4), using 15*21 thin shell elements (Fig. 1) with five degrees of freedom per node (three displacements and two in-surface rotations). This number of elements has been optimised to ensure computation convergence (disregarding stress concentrations in the corners).

3.2. Limit states for design

Eurocode 1: Actions on Structures [14] describes the serviceability (SLS) and ultimate (ULS) limit states which need to be fulfilled. The shells are analysed for the load cases defined in Eurocode 1 as if they spanned a closed building with the façades as separate structures and thus exerting no forces on the shell.

Eurocode 2: Design of Concrete Structures [12], limits the deflections under serviceability load combination to span/250. For the ultimate limit state, the shells have to comply with the maximum tensile and compressive material design stress criteria (Table 1). The considered load cases are self weight, wind and snow. After analysing the 2 m span doubly curved shell under all possible load combinations, two determining load combinations are selected: Load combination 1 (LC1), resulting in the highest downward load (self weight * 1.35 + asymmetrical wind pressure * 1.5 + asymmetrical snow * 1.5 * 0.5) and load combination 2 (LC2) resulting in the highest upward load (self weight * 1 + asymmetrical wind suction * 1.5; no snow).

4. Structural Analysis of the doubly curved 2 m span textile reinforced IPC shell

4.1. Analysis of the minimum shell thickness for 2 m span doubly curved IPC shell

The tensile stresses occurring in the 8 mm thick doubly curved 2 m span GTR-IPC shell (Table 2) remain below the design stress (20 MPa, see Table 1) under ULS load combination 1 and ULS load combination 2. For smaller thicknesses, the occurring stresses exceed the design stress.

For the 8 mm thick doubly curved 2 m span shell under self weight, the serviceability load combination, the maximum upward (0.065 mm, see Table 2) and downward (0.25 mm) deflections remain far below the serviceability limit (8 mm = 2000 mm/250), defined in Eurocode 2. Due to its double curvature, the shell under study has a high geometrical stiffness. For this 2 m span doubly curved shell, the ultimate limit state is the dimensioning limit state. The influence of material stiffness reduction due to repeated loading and the accompanying larger deflections are thus unlikely to have a significant effect on the minimum shell thickness.

Under the SLS load combination (self weight), tensile stresses remain very low (less than 1 MPa, see Table 2). Fig. 3 shows that the linear elastic material behaviour assumption is thus sufficient to obtain accurate results for the displacements in SLS.

Conclusively, for the 2 m span doubly curved glass textile reinforced IPC shell, a thickness of 8 mm fulfils both serviceability and ultimate limit states. Although the 2 m span shell represents a hypothetical case as practical shell dimensions would be larger, this case study still clearly shows that the thickness of a small span GTR-IPC shell is far inferior to the minimum thickness required for an equivalent steel-reinforced concrete shell (typically 70 millimetres).

4.2. Structural behaviour under ULS of 2 m span doubly curved shell

The largest stresses in the 2 m span shell (Table 2) occur under ULS load combination 1; Fig. 4 shows that this asymmetrical load combination of self weight, wind and snow leads to an asymmetrical stress distribution. At the leeward side of the shell, the upper surface is stressed in compression ($\sigma_{c,max} = -18.6$ MPa), and the lower shell surface undergoes tension ($\sigma_{t,max} = 18.6$ MPa). At the windward side of the shell, the upper surface experiences tensile stresses ($\sigma_{t,max} = 17.5$ MPa) while the lower surface has compressive stresses ($\sigma_{c,max} = -17.9$ MPa). The maximum stress values are almost equal in magnitude, but opposite at upper and lower surfaces. This determining ULS load combination thus creates globally two zones in the shell which are mainly transferring stresses in bending. The lever action turns out to be important for the asymmetric load combination-bearing capacity of this 2 m span doubly curved shell. As the textile reinforcement is placed close to the outer surfaces, the entire shell thickness contributes to the lever action required for bending and is small (thickness/span ratio equals 1/25).

5. Analysis of doubly curved 10 m span textile reinforced IPC shell

The doubly curved shell is given more realistic dimensions: 10 m span width and 5 m height. The shell has an increased thickness and thus increased self weight, but identical wind pressures and snow surface load as the 2 m span shell. The structure is evaluated under the expected determining ULS load combination 1 and verified under ULS load combination 2, as discussed for the 2 m span shell in Section 4. For a thickness of 40 mm, principal in-plane tensile stresses (Table 3) slightly remain under the design stress (20 MPa, see Table 1). The stress distribution pattern under ULS load combination 1 is similar to that of the 2 m span shell.

Even though the self weight has reached the same order of magnitude as the maximum value of the variable wind load, the resulting stresses under this uniformly distributed vertical loading are still much lower (see Table 3: LC 1 (18 MPa) versus self weight $*$ 1.35 (5 MPa)). This structure is more apt to uniformly distributed vertical loads than to asymmetrical load. A first design under self weight only, would therefore give a significant underestimation of the behaviour of the structure.

Under SLS load, the displacements (maximum 6.3 mm, see Table 3) are more than 6 times below the serviceability limit (40 mm = span/250), defined by Eurocode 2 and the tensile stresses remain in the linear elastic zone (lower than 2 MPa). The ULS thus also defines the dimensioning limit state for the 10 m doubly curved shell.

Finally, the minimum thickness equals 1/25 of the span, exactly the same ratio as for the 2×2 m shell. The following section derives the analytical relationship between the height and the span of a beam under a uniformly distributed load, and links this to the relationship between thickness and span of small span doubly curved shells.

6. Design rules for small span, doubly curved TRC shells

6.1. Beam under bending theory

For a simply supported rectangular beam (unity width, height t, span L and density ρ) in bending under a uniformly distributed load q (Fig. 5), the Euler–Bernoulli beam equation gives the following relationship (6.1) between maximum stresses occurring at the outer surface of the section experiencing the largest moment, and the span, beam height and uniformly distributed load:

$$\frac{qL^2}{t^2} \approx \sigma_{max} \tag{6.1}$$

Table 2
Maximum stresses and displacements under ULS and SLS load combinations for 8 mm thick doubly curved 2 m span GTR-IPC shell

	Max stresses upper surface (MPa)		Max stresses lower surface (MPa)	
	Tension	Compression	Tension	Compression
Ultimate limit state				
LC 1	17.5	−18.6	18.6	−17.9
LC 2	15.1	−17.5	18.2	−15.1
	Max. displacements (mm)		Max tensile stresses (MPa)	
	Downward	Upward	Upper surface	Lower surface
Serviceability limit state				
Self weight	0.25	0.06	0.25	0.15

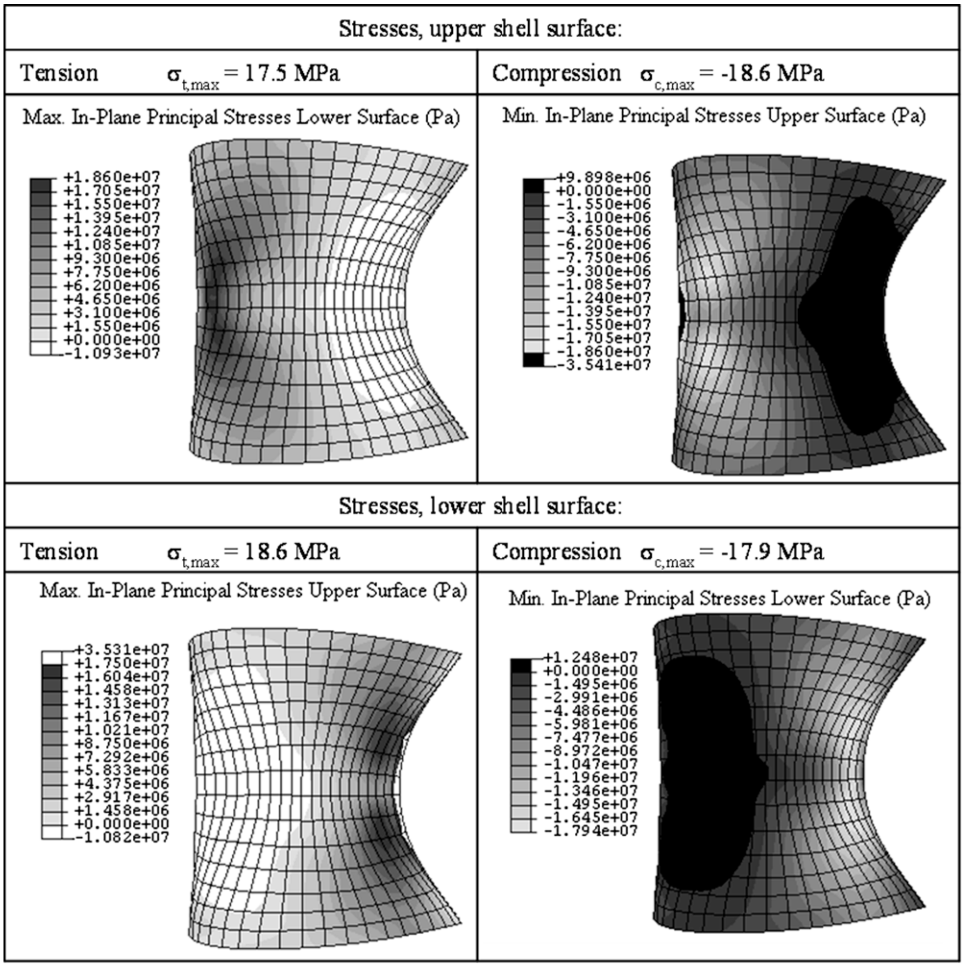

Fig. 4. Plan view: distribution of tensile and compressive stresses at upper and lower surface of 2 m span doubly curved GTR-IPC shell, under load combination 1.

Table 3
Maximum stresses and displacements under ULS and SLS load combinations for 40 mm thick doubly curved 10 m span GTR-IPC shell

	Max stresses upper surface (MPa)		Max stresses lower surface (MPa)	
	Tension	Compression	Tension	Compression
Ultimate limit state				
LC 1	18.0	−18.2	17.9	−18.7
LC 2	15.2	−17.1	17.5	−15.4
Self weight * 1.35	1.6	−4.3	1.0	−4.8
	Max. displacements (mm)		Max tensile stresses (MPa)	
	Downward	Upward	Upper surface	Lower surface
Serviceability limit state				
Self weight	6.3	1.7	1.3	0.7

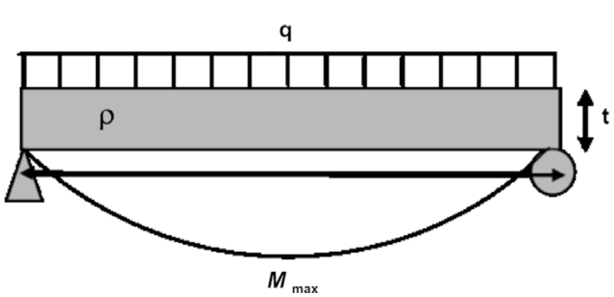

Fig. 5. Simply supported beam under uniformly distributed load *q*.

The load q accounts for the uniformly distributed loading superimposed to the dead load. Contrarily to the uniformly distributed loading, the dead load increases with the beam thickness. When the beam is loaded under self weight only, formula 6.1 can be written in terms of the constant density ρ:

$$\frac{\rho L^2}{t} \approx \sigma_{max} \tag{6.2}$$

For a statically determinate beam under self weight only, the necessary beam height t is proportional to the square of the span (6.2), contrarily to the linear proportional rule when self weight can be neglected (6.1). These equations demonstrate that in order to limit the maximum stresses at the outer surface by increasing the beam's thickness, it is most favourable to reduce the ratio (response to) self weight load to (response to) live loads.

6.2. Span-minimum thickness relationship

Comparison between the minimum thicknesses of the 2 m (t = 8 mm) and 10 m span (t = 40 mm) shells, shows that the thickness increases with the same factor as the span. This observation corresponds to Eq. (6.1), which represents the case where self weight is neglected. The shell thicknesses are determined by the variable asymmetric load combinations, provoking large bending stresses. In these load combinations, the effect of self weight is indeed negligible as shown in the different analyses. The doubly curved small span (up to 10 m) shells might work in bending under the dimensioning asymmetric load combinations, however the response under self weight load is so small relative to the response under variable loads, that its disadvantageous effect of increasing thickness with the square of the span is eliminated.

The proportional rule has been evaluated for a 15 m span doubly curved GTR-IPC shell. According to (6.1), the corresponding shell thickness for 15 m span is 60 mm and approaches the minimum thickness required for steel-reinforced concrete shells. FEM analysis shows that the 60 mm thick doubly curved 15 m span shell just fulfils both ULS and SLS. The response under self weight increases, but its contribution still remains negligible. Maximum allowed deformations, prescribed by Eurocode 2 [12], are more than four times higher than the deflections under SLS.

6.3. Thickness–maximum stress relationship

The analytical maximum stress–thickness relationship (6.1), neglecting self weight, is validated against the FEM results for the 2 m span doubly curved GTR-IPC shell. Fig. 6 shows that the maximum tensile and compressive stresses, at the upper and lower surface of the 2 m span shell under ULS load combination 2 are inversely proportional to the square of the shell thickness and thus show good agreement with Eq. (6.1).

7. Influence of textile reinforced concrete material properties on the design of small span doubly curved shells

The structural performance of the doubly curved 2 m span shell is studied for two different TRC composites being an alkali-resistant glass fibre textile Ordinary Portland Cement fine tuned mortar (GTR-OPC) and GTR-IPC. Table 1 gives the material properties for both glass fibre textile – cement combinations.

Maximum tensile stresses in the 8 mm thick, 2 m span shell exceed the GTR-OPC design strength (10 MPa, see Table 1) under the ULS load combinations. The minimum shell thickness can be determined using formula (6.1) as self weight has a low contribution to the response under dimensioning ULS asymmetric load combinations. Taking into account the reduced GTR-OPC design tensile strength (10 MPa, instead of 20 MPa for GTR-IPC), the minimum thickness of the 2 m span shell equals 11 mm.

Table 4 shows that maximum stresses in this 11 mm thick shell stay indeed below the GTR-OPC design strength (10 MPa in tension and 25 MPa in compression) under both ULS load combinations. Deflections of the shell under SLS are lower than for the GTR-IPC shell, as the material stiffness as well as the shell thickness is increased. These comparisons show that when using GTR-OPC as a structural shell material, dimensioning limit state is still the ULS. Formula (6.1) has thus proven to offer a suitable design rule for the 2 m span shell, when altering the material combination. Table 5 shows the minimum thicknesses of the 2, 10, and 15 m span GTR-OPC doubly curved shells, according to (6.1).

Formula (6.1) also allows one to adjust the material properties in function of the structural requirements. In the case of the shells under study, glass textile reinforced IPC containing high fibre volume fractions, can offer a more suitable solution than GTR-OPC when the main design criterion is slenderness. This slenderness ratio is only limited by the maximum fibre volume fractions that can be added to the IPC-matrix. Within manufacturing experimental programs, fibre volume fractions of 20% have been achieved up to now by hand lay-up or by industrialised impregnating means [15]. However, since according to (6.1) the minimum thickness of a doubly curved shell with span up to 15 m only increases with the root of the material design stress reduction the fibre volume fraction has a relatively modest influence on the slenderness.

8. Conclusions

This paper shows the main structural advantages of using TRC (textile reinforced concrete) over traditional steel-reinforced concrete in doubly curved shells with spans varying between 2 and 15 m: its tensile load bearing capacity, the resulting redundancy of steel reinforcement and corrosion cover and its capability to create thin, material efficient small span shells. Tensile stresses in the

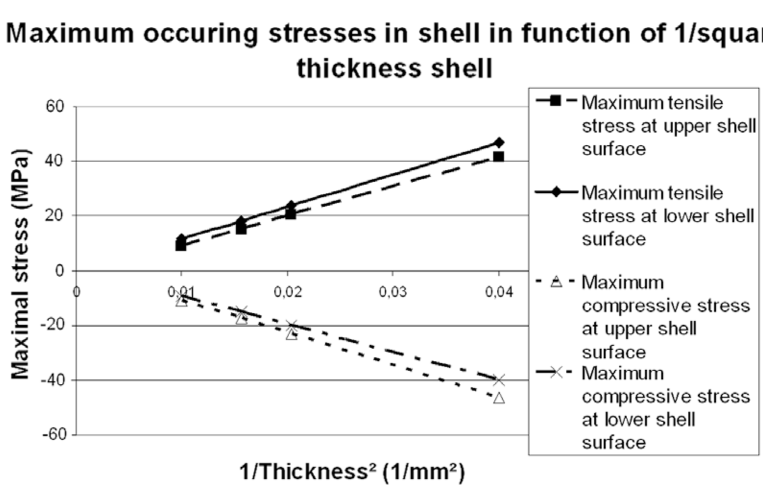

Fig. 6. Maximum tensile and compressive stresses at upper and lower surface of 2 m span doubly curved GTR-IPC shell under ULS load combination 2 in function of 1/square of shell thickness.

Table 4
Maximum stresses and displacements under ULS and SLS load combinations for 11 mm thick doubly curved 2 m span GTR-OPC shell

	Max stresses upper surface (MPa)		Max stresses lower surface (MPa)	
	Tension	Compression	Tension	Compression
Ultimate limit state				
LC 1	9.0	−9.2	9.6	−9.3
LC 2	7.3	−8.8	9.5	−7.7
	Max. displacements (mm)		Max tensile stresses (MPa)	
	Downward	Upward	Upper surface	Lower surface
Serviceability limit state				
Self weight	0.10	0.01	0.21	0.12

Table 5
Comparison of the minimum thicknesses of GTR-IPC and GTR-OPC doubly curved shells, with 2 m, 10 m and 15 m span width

Span width doubly curved shell (m)	Minimum shell thickness (mm)	
	GTR-IPC	GTR-OPC
2	8	11
10	40	60
15	60	90

doubly curved shells, occurring under dimensioning load combinations of self weight, wind and snow, are of the same order of magnitude as the compressive stresses, and can be carried by TRC. Bending moments that occur in the doubly curved shells under asymmetric load combinations can efficiently be carried. Since the corrosion cover is redundant for TRC and textile reinforcement is uniformly distributed over its thickness, the whole shell thickness contributes to the lever action needed to resist bending. In conclusion, the application of TRC in doubly curved small span shells leads to reduced shell thickness and a minimum of dead weight in comparison to steel-reinforced concrete shells. For example, the 10 m span GTR-IPC doubly curved shell fulfils all limit states with a minimum thickness of only 40 mm (in comparison with a traditional steel-reinforced concrete shell of minimum 70 mm).

Due to the high geometric stiffness of chosen shell form, deformations under SLS load combination remain far below the limit. Stresses due to asymmetrical ULS combinations have proven to be the determining factor for the design of these structures. The shell self weight hardly contributes to the ULS stresses. Comparison of the doubly curved shell designs to the beam theory (for a statically determined beam in bending under a uniformly distributed loading and neglecting self weight) led to the analytical derivation of a simple preliminary design rule for small span TRC shells (2–15 m).

The results of this study are promising for the application of TRC in free form and/or small span shells. First of all, the flexible reinforcement facilitates manufacturing of highly curved creative forms. Secondly, TRC has proven to efficiently carry tensile and compressive stresses due to membrane and bending action. The form freedom of TRC shells is thus not restricted to shell shapes working mainly in compression.

Acknowledgements

The Fund for Scientific Research in Flanders, Belgium (FWO) financially supports this research by the Ph.D. program of IR. Tine Tysmans and the post-doctoral position of Dr. Heidi Cuypers.

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