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STRUCTURAL OPTIMIZATION OF A THIN-SHELL BRIDGE STRUCTURE

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ABSTRACT

This paper demonstrates the use of topology optimization as a design tool for a thin-shell bridge structure. The presented topology optimization algorithm computes the material distribution within the shell while maximizing its overall stiffness for a given volume of material. The optimization routine is coupled to a Finite Element Method and finds its solution using the Fixed Point Iteration method or Picard Iterations. Besides obtaining the optimal shell thickness distribution, the topology optimization routine also suggests the optimal shape. The optimal shape enhances the mechanical behavior of the structure. The results of this study show that after topology optimization, the deck's deflection, the shell's Von Mises stresses, the eigenfrequency of free vibration as well as the deck's bending moments are improved. This study uses a simplified model of the existing steel shell of the Knokke Footbridge as a case study.

Keywords: *design tool, thin shell, topology optimization, finite element, Picard iterations, Reissner-Midlin, potential energy, thickness distribution*

1. INTRODUCTION

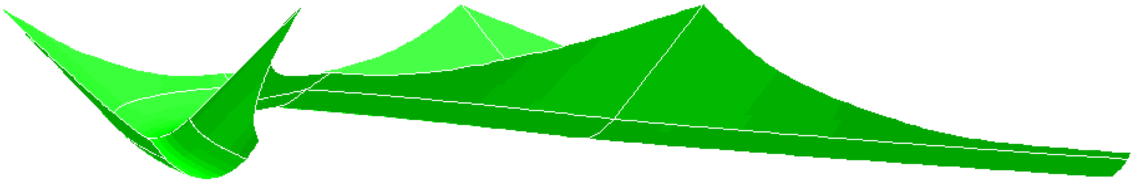
This work demonstrates the value of computational methods as a design tool to optimize a thin-shell bridge structure. A typical topology optimization problem consists of distributing a given amount of material in a design domain subject to load and support conditions, such that the stiffness of the structure is maximized.

Such topology optimization problems have previously been tackled. F. Belblidia and S. Bulman use a hybrid topology optimization algorithm which combines mathematically rigorous homogenization with intuitive methods [1]. R. Ansola and J. Canales have also implemented a computational method that integrates both shape and topology optimization [2][3]. Furthermore, T. Kimura and H. Ohmori also propose optimization methods for both shape and topology of shell [4].

Our approach focuses on the intuitive interpretation of topology optimization results in order to enhance the shape of a structure. The geometry of the existing 102 *m* span steel Knokke Footbridge [5] offers the basic static and dynamic model for this research. The footbridge has two 26 *m* approaching spans and a central span of 50 *m*. The structure is a steel thin shell with a reinforced 3 *m* wide concrete structural deck. Figure 1 shows the initial topology of the steel shell without the deck.

The method aims at finding the optimal thickness distribution throughout the shell to improve the structural performance (being the deck's deflection, the shell stresses, the deck's bending moments and the shell's dynamic behavior based on the eigenfrequency of free vibration).

Figure 1. Thin-shell structure: base geometry to be optimized with topology optimization



2. TOPOLOGY OPTIMIZATION THEORY

2.1 The minimum compliance problem

Topology optimization relies on the minimum compliance problem, also known as maximum global stiffness problem [6] [7].

Let U be the space of kinematically admissible displacements and E_{ad} the set of admissible elasticity tensor. The internal virtual work of an elastic body can be expressed as

$$a_E(\underline{u},\underline{w}) = \int_\Omega \underline{\underline{\varepsilon}}(\underline{u}) : \underline{\underline{\underline{\underline{E}}}} : \underline{\underline{\varepsilon}}(\underline{w})\, dV \qquad \forall\ \underline{w} \in U \quad (1)$$

where $\underline{\underline{\varepsilon}}$ are the linearized strains, $\underline{u}$ the displacement at equilibrium, $\underline{w}$ an arbitrary virtual displacement, and $\underline{\underline{\underline{\underline{E}}}} \in E_{ad}$ the elasticity tensor. The use of the index E indicates that the bilinear form a_E depends on the design variable.

Let ρ be the non-dimensional thickness of the shell ie the material distribution function within the domain Ω. ρ is the design variable of our problem. The external virtual work can be written as

$$l_E(\underline{w}) = \int_\Omega \underline{f}(\rho).\underline{w}\, dV + \int_\Gamma \underline{t}.\underline{w}\, dS \qquad (2)$$

where $\underline{f(\rho)}$ are the body forces within the domain Ω and $\underline{t}$ the tractions applied to the boundary Γ. The use of the index E indicates that the external virtual work depends on the design variable ρ. The minimum compliance problem can be expressed as :

$$\begin{cases} \displaystyle\min_{E \in E_{ad},\, \underline{u} \in U} \quad l_E(\underline{u}) \\ \text{subject to} \quad \begin{cases} \dfrac{1}{2} a_E(\underline{u},\underline{w}) = l_E(\underline{w}) \quad \forall\ \underline{w} \in U \\ \int_\Omega \rho(\underline{x})\, d\Omega = V \\ \rho_{\min} < \rho < \rho_{\max} \end{cases} \end{cases}$$

(3)

The principle of minimum potential energy enables equation (3) to be expressed using the max-min formulation:

$$\begin{cases} \displaystyle\max_{E \in E_{ad}} \quad \min_{\underline{u} \in U} \quad \left\{ \tfrac{1}{2} a_E(\underline{u},\underline{u}) - l_E(\underline{u}) \right\} \\ \text{subject to} \quad \begin{cases} \int_\Omega \rho(\underline{x})\, d\Omega = V \\ \rho_{\min} < \rho < \rho_{\max} \end{cases} \end{cases} \qquad (4)$$

2.3 Thin-shell structures and Finite Element Method

A thin-shell structure is defined as a shell with a thickness t which is small compared to its other dimensions. The thickness at location $\underline{x}$ is related to the distribution function ρ by the following relation:

$$t(\underline{x}) = t_0.\rho(\underline{x}) \qquad (5)$$

where t_0 is the reference thickness.

Expression (4) can be discretized using the Finite Elements Method. The shell is modelized using the Reissner-Midlin model for plates and curved structural elements [8]. The material distribution function ρ becomes constant inside each element. The minimum compliance problem expressed in expression (4) can be stated as:

$$\begin{cases} \displaystyle\max_{[\rho]} \quad \min_{[d]} \quad \tfrac{1}{2}[d]^t[K(\rho)][d] - [d]^t[f(\rho)] \\ \text{subject to} \quad \displaystyle\sum_e \rho^e v^e = V \end{cases} \qquad (6)$$

where $[\rho]$ are the unknown thickness coefficients, $[d]$ are the unknown nodal displacements, $[f]$ are the nodal loads, v^e is the volume of element *e,* and $[K(\rho)]$ is the global stiffness matrix defined as the usual assembly $\underset{e}{A}$ of all the element stiffness matrices $[k^e(\rho^e)]$:

$$[K(\rho)] = \underset{e}{A}\left[k^e(\rho^e)\right] \qquad (7)$$

According to the Reissner-Mindlin shell theory, the stiffness contribution to the shell is made up of “plate” and “membrane” contributions. Its “plate” contribution matrix is related to its thickness t by

$$\left[k^e(t)\right] = \frac{t^3}{12}\left[k^e{}_{bending}\right] + t\left[k^e{}_{shear}\right] \qquad (8)$$

where $\left[k^{e}{}_{shear}\right]$ and $\left[k^{e}{}_{bending}\right]$ are the respective element stiffness matrices which are consequences of shear and bending. The global stiffness matrix is not a linear function of the thickness but a function of the thickness cubed. An important consideration is the number of shear strain constraints engendered in the thin plate limit (i.e. as $t \rightarrow 0$) creating "shear locking". These constraints are alleviated by using four node quads and reduce quadrature on the shell shear stiffness [8].

Formulating a thickness distribution problem for topology optimization can lead to mesh dependent solutions. It is a common problem in plane elasticity and plate problems, as continuous design variables are driven to the upper and lower bounds, resulting in a 0-1 topology. However, the curved geometry and importance of the shear and membrane stiffness terms to this shell structural stiffness leads to optimal thicknesses ranging between the lower and upper bounds. As the solution may thus be considered continuous and not 0-1, we do not have issues of mesh dependency (nor checkerboard patterns).

2.4 The Fixed Point Iteration method

The optimization problem is solved using the Fixed Point Iteration method, also known as the Picard iterations. These iterations are accomplished by fixing one set of variables while varying the other set. The fixed set of variables is then updated based on the changes made to the other set of variables and the process is repeated until the problem converges to a solution.

For minimum compliance problems, the Fixed Point Iteration approach is mathematically equivalent to the so-called adjoint approach frequently used in standard topology optimization. The adjoint approach also computes displacements for given design and uses these displacements in the sensitivity and design step computations. The minimum compliance problem is self-adjoint, leading to sensitivities that are a function of strain energy - exactly the same expression as obtained when differentiating this paper's maximization objective function. The shell in our paper is modeled as a doubly curved shell in 3D as detailed in Hughes [8] Chapter 6 for which there is no easy derivative with respect to thickness. Therefore, we compute sensitivities by finite differencing. Further, this allows us to have a general tool not connected to any specific finite element formulation. The optimization starts with an initial uniform thickness distribution $[\rho]$. The corresponding stiffness matrix $[K]$ is calculated. Then, the displacements $[d]$ are computed by using the equilibrium equation $[K][d]=[f]$ and fixing $[\rho]$ (solving the equilibrium equation is the same as minimizing the energy over $[d]$). Subsequently displacements $[d]$ are fixed to compute the distribution $[\rho]$ from the maximization of the energy. The stiffness matrix is updated with the new distribution $[\rho]$ and the equilibrium equation is then no longer satisfied. A series of fixed point iteration has to be completed until the values for $[\rho]$ and $[d]$ both satisfy the equilibrium equation. The topology optimization algorithm is illustrated in Figure 2.

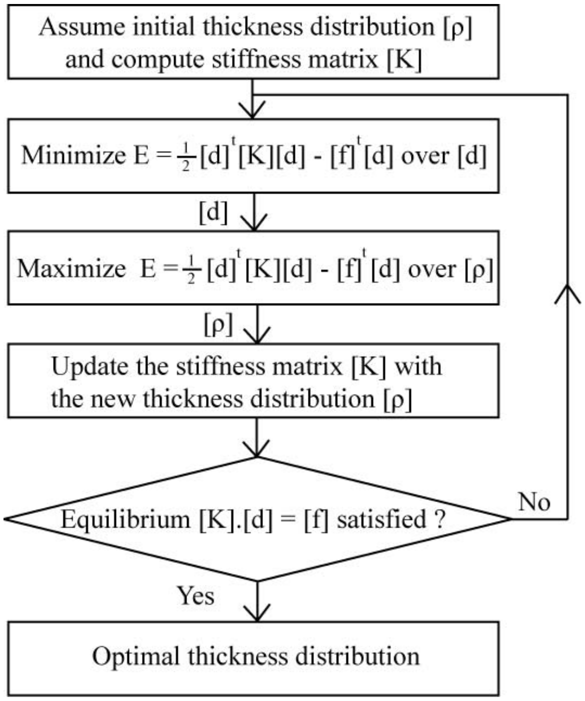


Figure 2. *Flow diagram of topology optimization algorithm*

The maximization problem over $[\rho]$ is solved using the commercial software package *Snopt* [9]. The method of "Move Limits" is chosen as the solution method due to the cubic non-linearity of the maximization problem (see equation 8). "Move Limits" enables $[\rho]$ to vary by limited amounts for each iteration. These limits are kept constant. Furthermore, the objective function to be optimized (eq. 6) may have many local extrema. Thus there is no guarantee to find the global optimum. For this reason, the initial uniform thickness distribution can be changed to an initial random thickness distribution. In this case study, the topology optimization algorithm converged to the same solution. This fact showed that the overall optimum was probably found, although it does not prove it.

3. TOPOLOGY OPTIMIZATION OF THE KNOKKE THIN-SHELL STRUCTURE

3.1 The initial solid shell

The aim of the study is to use the topology optimization routine as a design tool for the shell shown in Figure 1. This shell has the same general shape as the realized Knokke Footbridge [5]. The shell mesh is created using Femgen [10]. The shell is modeled as a doubly curved shell in 3D as detailed in Hughes [8] Chapter 6. The mesh is made of four-node quad elements.

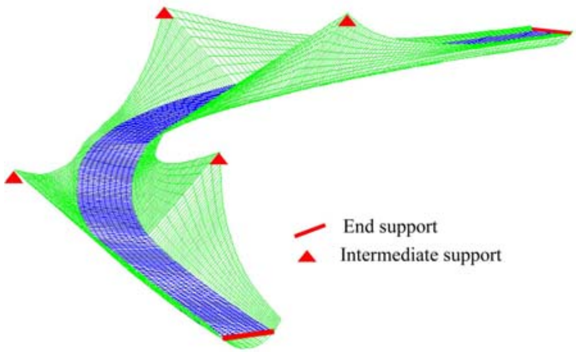


Figure 3. *The deck (blue) lies in the thin-shell structure (green)*

The shell sits on six supports: two end supports (roller) and four intermediate supports (using masts). The masts are not part of this optimization study. The 3 m wide deck lies in the thin-shell structure as shown in figure 3. The deck is not part of the thickness optimization but it contributes mainly to the load of the structure and its global stiffness. The mass density of the deck is *2,5* $kg.dm^{-3}$ (regular concrete) and its overall weight is 12,5 MN. The shell is made of steel with a mass density of *7,8* $kg.dm^{-3}$. The thickness of the original shell varies from *10 mm* to *30 mm* and the average thickness is *13,4 mm.* The shell is optimized in function of a combination of self weight (deck and shell) and variable load.

3.2 Topology Optimization

The presented Topology Optimization distributes the thickness throughout the shell to maximize the overall stiffness under self weight and variable load. The optimization is performed with the Finite Element analysis program *Dynaflow* [11]. As expressed in equation (5), the thickness t^e of each shell finite element is found by multiplying the reference thickness t_0 (in this case chosen to be $t_0 = 15mm$) by the thickness distribution function ρ. The bounds of the distribution function ρ are set to $\rho_{min} = 0.2$ (thus $3mm$) and $\rho_{max} = 2.0$ (thus $30mm$). Furthermore, the overall volume constraint for the entire structure is imposed through the mean thickness ρ_{mean},The simulations are done with different mean densities, varying from 0.2 to 1.8, with a step of 0.1. Figure 4 shows the optimal thickness distribution in the shell for different values the mean thickness ρ_{mean}.

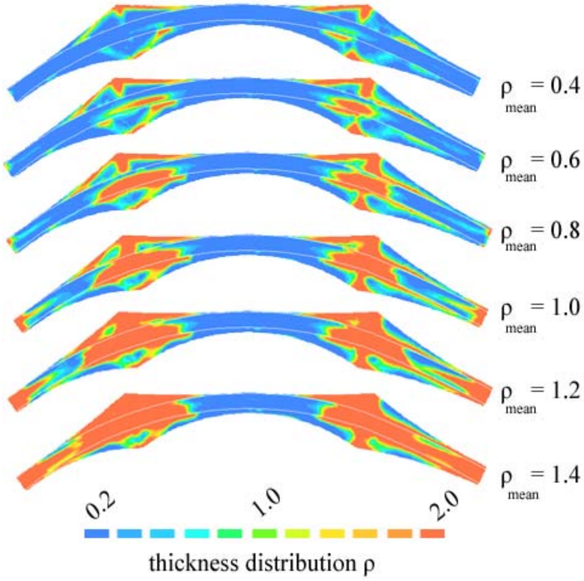


Figure 4. *Topology Optimization of the shell for different values of mean density* ρ_{mean}

Figure 5 shows the impact of Topology Optimization on the compliance of the structure. The thickness of the optimized shell varies between $3mm$ and $30mm$. These thicknesses are theoretical values. For practical bridge construction considerations, the minimum value of 3*mm* is not a realistic steel shell thickness. However this shell is much more efficient in terms of weight and stiffness than the shell with uniform thickness distribution. Figure 5 shows that the compliance of shell with the uniform thickness distribution decreases with the thickness showing that a thicker shell means a stiffer structure. As far as the optimized distribution is concerned, the compliance is convex with ρ_{mean} and optimal for a mean thickness ρ_{mean}=0.7. Figure 5 clearly shows that beyond this point, an increase of mean thickness, ie weight, reduces the global stiffness of the structure.

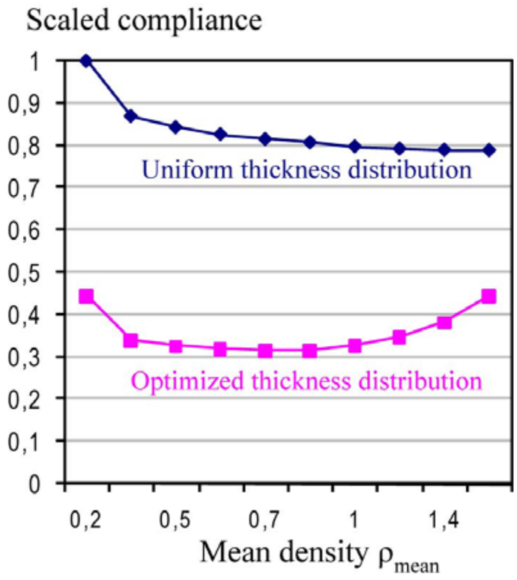


Figure 5. *Compliance minimization for a shell with uniform thickness (blue graph) and for a shell with optimized thickness distribution (pink graph)*

Figure 6 shows a close-up of the results of the Topology Optimization ($\boldsymbol{\rho_{mean}}$=0.7) for the area around the intermediate supports. This figure suggests the optimal location for openings and also shows where the thickness has to be increased. The shell with suggested openings is further investigated in the next section.

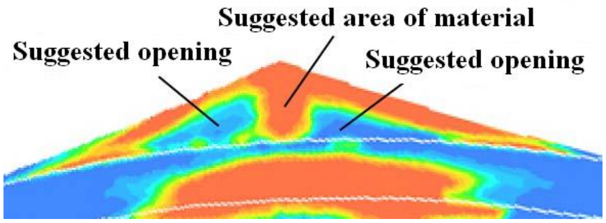


Figure 6. *Shape and location for openings in the solid shell suggested by the Topology Optimization routine.*

4. COMPARISON OF SHELLS

This section compares the static and dynamic behavior between the optimal topology of shell with openings and a simplified model of the realized Knokke shell. The two shells are presented in section 4.1 and 4.2. Comparison is conducted in section 4.3

4.1 The optimal topology of a shell with eight openings

The optimal topology of the shell with 8 openings is defined as the new shell generated after the interpretation of topology optimization results and its suggestions for eight openings as shown in Figure 6. There is no threshold value for removing material but rather and intuitive interpretation of Fig. 6. The shape given to the optimal topology of shell is shown in Figure 7, a close-up of its thickness distribution around the intermediate supports in figure 8.

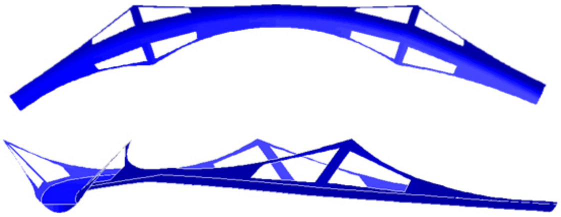


Figure 7. *Shape of the optimal topology of shell with eight openings near the intermediate supports.*

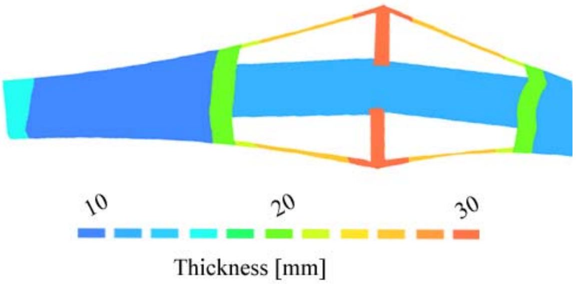


Figure 8. *Thickness distribution in the topology optimized shell with eight openings*

4.2 The model of the built shell of the Knokke Footbridge

The studied Knokke shell is a simplified version of the realized Footbridge built in Knokke. This shell has four openings around the intermediate supports, as shown in Figure 9. The thickness distribution, as built, is shown in Figure 10.

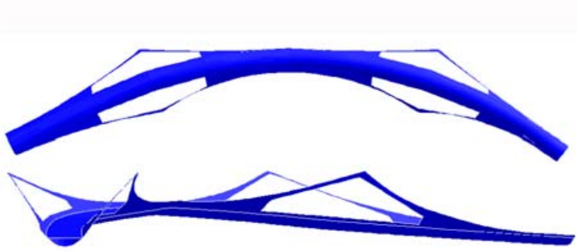


Figure 9. *Shape of the built Knokke shell*

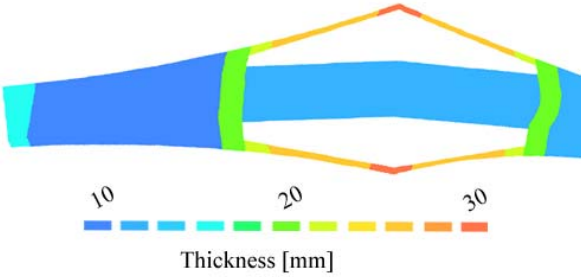


Figure 10. *Thickness layout in the Knokke shell, as built*

4.3 Comparison of the static and dynamic behavior of the shells

This section compares the structural behavior of the optimal topology shell with eight openings and the model of the built shell in Knokke. The structural behavior comparison criteria are 1. the deck's deflection in the vertical direction, 2. the shell's Von Mises stresses 3. the shell's bending moments and 4. dynamic behavior studies through the eigenfrequency of free vibration.

The vertical shell deflections are considered under a uniformly distributed variable load of 5 kN.m^{-2} according to :

prEN 1991-2 Euro code 1: Actions on structures – Part 2 – Traffic loads on bridgesNBN B03-101 Belasting van Bouwwerken: Wegbruggen.

The deflection is shown in Figure 12 for the optimal topology shell with eight openings and the model of the realized Knokke shell. The respective maximum deflections are not located at the same position. The maximum deflection is 77 mm for the Knokke model and 56 mm for the model with eight openings. The deflection for this last shell is 27% less than the realized shell. In both cases, the deflection remains under the service ability limit which requires the deflection to be less than *L / 400* (where *L* is the bridge span). In this case, *L / 400* equals 250 mm.

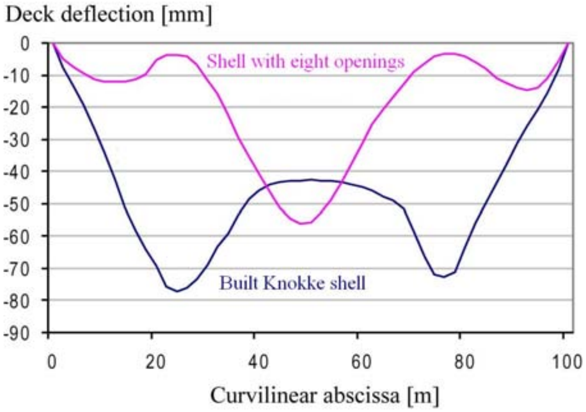


Figure 12. *Comparison of the deck deflection in the built Knokke shell and the optimal topology shell with eight openings*

The next criterion of comparison is the Von Mises stresses under the load combination of self weight and variable load. The maximum Von Mises stresses in the Knokke shell are 232 MPa (location shown in Figure 13). As far as the optimal topology shell with eight openings is concerned, the maximum occurring stress is 153 MPa (location shown in Figure 14), which is 34 % smaller than the stresses occurring in the built Knokke shell. In both cases, the stresses remain under the factored yield strength of steel used in the existing model (355 MPa * 0.7). The optimized shape reduces the stresses in the structure substantially but stresses the material less efficiently far below the factored yield strength . This observation can be explained through the fact that the topological optimization routine optimizes for global stiffness not strength.

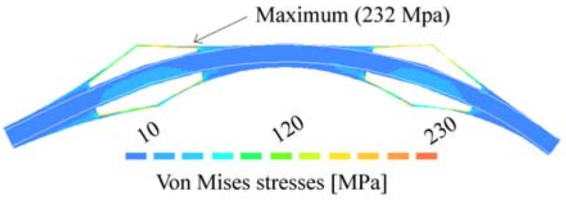


Figure 13. *Von Mises stresses in built Knokke shell under load combination of self weight and variable load*

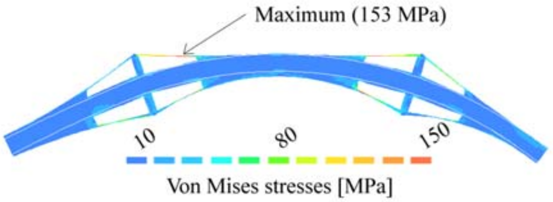


Figure 14. *Von Mises stresses in the optimal topology of shell with eight openings under load combination of self weight and variable load*

The next criterion is the bending moments in the deck, under combined self weight and variable load. As shown in Figure 15, the amplitude of the bending moment in the Knokke model is 19.1 kNm/m whereas it is 12.9 kNm/m for the optimized model.

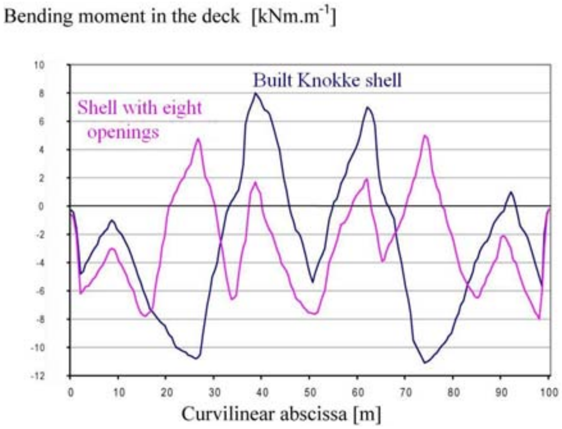


Figure 15. *Comparison of the bending moment in deck in between built shell and optimal topology of shell with eight openings*

The last criterion of comparison is the dynamic behavior of the structure studied through the eigenfrequency of free vibration according to the Belgian norm NBN B52-001. The first eigenfrequency of the Knokke model is 1.27 Hz. The first eigenfrequency of the optimal topology model is 0.92 Hz. Both shells stay out of the proscribed range of [1.7 Hz , 2.5 Hz] (NBN B52-001). This comparison shows that the presented topological optimization routine creates the stiffest possible shell with the least amount of material and with good dynamic behavior.

CONCLUSION

Topology optimization provides a powerful tool for thin shell design. Based on the maximum stiffness criterion, this numerical method generates the optimal thickness distribution within the shell. Furthermore the algorithm suggests adaptation of the new shape through including the location of openings which improve the shell's structural performance.

The optimal topology of shell is optimized for stiffness and statically and dynamically performs well above the established structural criteria laid out in the Codes. The basic model for the less stiff Knokke model also fulfills all stiffness, strength and stability requirements.

Structural design always fits between different objectives of stiffness, strength and stability. The optimal shape designed for overall stiffness may not be optimal for strength and subsequently not be the one that designers are looking for. This study illustrates this observation when it comes to creating the stiffest structure for a given amount of material. The realized Knokke shell design fulfills all structural criteria efficiently and has an elegant, poetic and sail-like appearance.

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