Proceedings of the International Association for
Shell and Spatial Structures (IASS) Symposium 2013
„BEYOND THE LIMITS OF MAN”
23-27 September, Wroclaw University of Technology, Poland
J.B. Obrębski and R. Tarczewski (eds.)

The Ribbed Floor Slab Systems of Pier Luigi Nervi

Allison B. Halpern[1], David P. Billington[2], Sigrid Adriaenssens[3]

[1]PhD Candidate, Civil and Environmental Engineering, Princeton University, Princeton, USA, *abhalper@princeton.edu*
[2]Professor Emeritus, Civil and Environmental Engineering, Princeton University, Princeton, USA, *billington@princeton.edu*
[3]Assistant Professor, Civil and Environmental Engineering, Princeton University, Princeton, USA, *sadriaen@princeton.edu*

Summary: This paper presents an historical and analytical evaluation of the ribbed floor slab systems developed by the Italian structural artist Pier Luigi Nervi (1891-1979). The historical discussion includes the evolution of the floor slab from a non-structural element to an inspired structural system, culminating in Nervi's patented ribbed floor systems. While the isostatic inspiration for the rib patterns of Nervi's floor systems is well-documented, the method used to generate these patterns is considerably unknown by comparison. The methods of experimental stress analysis identified by Nervi and the mathematical theories available prior to the 1949 isostatic floor system patent are discussed to clarify the means used to generate the isostatics. An Isostatic Line Tool developed for this paper is used to evaluate the correlation between the isostatics and Nervi's arrangement of ribs for the Gatti Wool Factory, Palace of Labor, and Large Sports Palace floor systems. This tool can be used by designers in the conceptual design phase to develop ribbed floor systems inspired by Nervi.

Keywords: *Nervi, ribbed slabs, isostatics, floor systems, computational tool*

1. INTRODUCTION

1.1. Evolution of Concrete Floor Systems

Prior to the 20th century, the prevalent materials used for floor systems were timber, masonry, brick, and tile. Safety concerns arising from several 19th century building fires and aspirations to construct taller buildings provided impetus for engineers to develop stronger, noncombustible floor systems. The absence of a universal system prompted the rapid filing of patents to protect the proprietary nature of these new systems [1].

Despite the development of reinforced concrete in the latter half of the 19th century, concrete was first used for its fireproofing rather than its structural qualities. The 1844 Fox and Barrett floor system, patented in the UK by Fox, was the first to use concrete as a fireproof covering over timber planks and cast-iron joists [2]. Though the first floor system to use concrete in a structural capacity was patented in 1854 by Wilkinson in the UK, it was Monier's 1873 French patent that stimulated the spread of structural concrete floor systems. Monier experimented with the layout of iron reinforcement within concrete floors. The widespread use of the Monier system was due to the financial sponsorship of the German firm of Wayss and Freytag, who in 1885 obtained the rights to the Monier patent [1].

The first reinforced concrete framing and floor system was patented by Hennebique in 1892. The success of this system in France provided his firm with the financial impetus to further develop and promote the system internationally. Although the Monier and Hennebique systems provided the fire resistance and structural capacity required for taller construction, the orthogonal arrangement of ribs essential to these systems produced an imitative timber joist and beam aesthetic [2].

1.2. Cement-based Floor Developments in Italy

In 1883, Monier filed a series of Italian patents, which included applications for floor systems. However, the development of reinforced concrete floor systems in Italy was soon hindered by a nearly decade long economic crisis. In 1892, Hennebique filed an Italian patent application for his reinforced concrete floor and framing system, followed five years later by an updated version with improvements for beam reinforcement. While the first major projects using this system could not begin until the start of the economic recovery in 1898, the spread of this system throughout Italy was largely due to Hennebique's marketing expertise. Not only did Hennebique promote the new system as immune to fire and both lighter and cheaper than a comparable iron system, he also appointed local engineers as agents of the patented system authorized to promote its use in new construction projects [3].

The first native Italian patent related to reinforced concrete was filed by Carlo Poma in 1893, which improved upon the preexisting Monier patent by providing a cheaper, more workable alternative through modifications of the aggregate proportions [4]. The economic recession not only slowed cement sales but also deterred the industrialization of the construction site, which retained the "artisanal" masonry methods of construction. The rise of Hennebique's reinforced concrete system coupled with the artisanal construction site stimulated the integration of masonry elements into concrete floor systems. The combination of hollow bricks (pignatta) and cement in concrete floors resulted in a lighter structural system with improved material economy; the first patent for a pignatta and concrete floor system was filed by Sigismondo Ghilardi in 1902 [5].

Due to the destruction caused by the 1908 Messina and Reggio Calabria earthquake, an international competition was established by the Società Cooperativa Lombarda di Lavori Pubblici in 1909 to establish the best material for withstanding earthquakes. Arturo Danusso (1880-1968), a practicing Italian engineer and later a professor at the Polytechnic of Milan in strict relation with Nervi his entire life, received highest recognition in the competition. Danusso ascertained that reinforced concrete structures would provide the most reliable resistance to earthquakes, which established reinforced concrete as the preeminent construction material for Italy's seismically-active regions [6]. However, when Italy declared war in 1915 construction plummeted due to the ensuing supply and labor shortages. Postwar reconstruction saw reinforced concrete restored as the main construction material of Italy [7], which led to the development of new building regulations in 1927. This first update in 20 years prompted the development of several references with practical calculation methods for designing and constructing reinforced concrete structures [3].

After graduating from the University of Bologna in 1913, Pier Luigi Nervi worked for the Società Anonima per le Costruzioni Cementizie (SACC), which was owned by Attilio Muggia, one of Nervi's professors and mentors at the University. As Muggia had obtained the rights to the Hennebique patent for central Italy in 1895, Nervi was exposed to the avant-garde of reinforced concrete construction early in his career [8]. Excluding a hiatus during WWI, Nervi worked at SACC from 1913 to 1923, after which he founded his own firm, Soc. Ing. Nervi & Nebbiosi, with Rodolfo Nebbiosi. In 1932, Nervi transitioned Soc. Ing. Nervi & Nebbiosi into a second company, Soc. Ing. Nervi & Bartoli, which allowed Nervi to control both the design and construction of a project [9]. Shortly thereafter, the Italian Fascist political atmosphere shifted into a state of Autarchy, or self-sufficiency. As metal reinforcement was primarily imported from foreign manufacturers, reinforced concrete quickly became an Anti-Autarchic material [10]. This prohibitive designation was a radical change from reinforced concrete construction symbolizing the architecture of the regime [3].

When reinforcement was first partially and then fully banned in 1935 and 1939, respectively, engineers were forced to use either traditional masonry arch systems or experiment with atypical reinforcement materials manufactured within the Kingdom of Italy. As cement was a material produced in mass quantities in Italy, this new research was characterized by cement-based floor systems with nominal reinforcement, no reinforcement, or reinforcement made with materials other than iron or steel, e.g., bamboo, aluminum. Patents of floor systems with a drastic reduction or complete elimination of reinforcement were filed throughout Italy. One of the most promising systems, the S.I.F. (Senza Impiego di Ferro, without the use of iron) floor, was patented by Eugenio Miozzi in 1937. To compensate for the absence of the required tensile capacity typically provided by reinforcement, the S.I.F. floor was composed of several layers of terracotta tile joined with high strength cement plaster and applied to the extrados and intrados of a traditional pignatta and concrete rib floor. While structurally sound, the S.I.F. floor was too time-consuming and labor-intensive for widespread use [5].

During the partial reinforcement ban, Nervi started on a series of hangars for the Italian Air Force, the first series of which could only be made with timber formwork and concrete poured on-site. This expensive, time-consuming construction process provided impetus for Nervi's 1939 patent for an efficient, economical construction procedure, which used prefabricated concrete elements. Nervi also focused on developing a new material to facilitate the consideration of fluid structural forms. This material exploration resulted in the creation of ferrocement, which is comprised of layers of flexible wire mesh encased in sufficient cement to achieve complete coverage at a reduced thickness [11]. In the years prior to his 1949 isostatic ribbed floor patent, Nervi explored a means of using ferrocement in tandem with his 1939 patented construction procedure [12]. The extension of the 1939 patent to include mass-produced, reusable ferrocement forms on moveable scaffoldings made constructing large floors supported by evenly spaced columns fiscally possible.

Fig. 1. Tobacco Factory Floor System [13]

Nervi first tested this moveable formwork procedure (entitled the "Nervi System") with the construction of the Bologna Tobacco Factory in 1949 (Figure 1). While the formwork produced a pattern imitative of the traditional beam and joist systems, the successful implementation of this construction procedure inspired Nervi's departure from orthogonality to the curved forms of the isostatic ribs. Although the isostatic rib patterns are considered integral to the "Nervi System," the concept of aligning floor ribs along the isostatics of the principal bending moments is attributed to Aldo Arcangeli, a practicing engineer who worked in the office of Soc. Ing. Nervi & Bartoli [12]. Arcangeli is noted as the author of the invention listed in the corresponding 1949 patent (No. 455,678), filed by Soc. Ing. Nervi & Bartoli, which included the improvement in the construction of a variety of structures, e.g., floors, domes, by the arrangement of support ribs along the isostatics of bending moments or normal stresses [14].

Arcangeli determined from classical plate theory that a 2D continuous body subjected to normal forces produces two families of orthogonal curves (isostatics), tangential to the principal bending moment trajectories, along which torsional moments are equal to zero. If this continuous body is replaced by ribs oriented along the isostatics, then the rib structure and the continuous body would have identical structural behavior under identical loading and support conditions [14]. Nervi drew inspiration from the patterns formed by the isostatics; he stated: "it was amazing to find that by thus limiting our task to the interpretation of a purely physical phenomenon, we were able to discover unexpected and expressive new forms [12]," and asserted that "in harmony with such inspiration, reinforced concrete beams lose the rigidity of wooden beams or of metal shapes and *ask* to be molded according to the line of the bending moments" [13].

2. NERVI'S METHOD FOR EVALUATING ISOSTATICS

While the structural behavior on which the isostatic rib patterns are based has been highlighted by Nervi, the manner in which these patterns have been generated has seen limited literary discussion. This section explores the experimental and theoretical analysis methods emphasized in Nervi's writings and the application of these analysis techniques to generating isostatics.

2.1. Strain Gauge Methods

In *Structures*, Nervi identifies two domains of experimental stress analysis: strain gauge methods and photoelasticity. Strain gauge methods rely on devices capable of measuring strain via mechanical, optical, electrical, acoustical, and pneumatic methods, to determine the displacements and stresses at points on a small-scale model [15]. Nervi first used mechanical strain gauges to determine the magnitude of the stresses in the ribs of a scale model for the 1935 Orvieto hangars [13]. In the late 1930s, Simmons and Ruge independently developed bonded-wire electrical-resistance strain gauges [15], which permitted the calculation of principal stress directions.

To find the stress field on the surface of a flat slab, it is necessary to use three-element strain gauge rosettes. These rosettes include three strain-gauges, each oriented at a different angle relative to the two in-plane axes (x and y), which provide three strain measurements corresponding to the three orientation angles. Using strain-transformation equations, the three Cartesian components of strain (ε_{xx}, ε_{yy}, and γ_{xy}) can be calculated and can be used to find the principal strain direction at the measurement location [15]. While calculating the principal stress directions from the rosette readings is simple, the experimental preparations and procedure are costly and time-consuming. Several rosettes would be needed to obtain enough data to clearly represent the full field of principal stress trajectories. As this method does not provide a more accessible way to obtain isostatics than theoretical calculations, strain gauge methods could not have been Nervi's initial means of finding isostatics.

2.2. Photoelasticity

Nervi's fascination with stress visualization in photoelasticity experiments (led by Danusso at the Polytechnic of Milan) suggests Nervi's possible use of photoelasticity to generate isostatics [12].

Photoelasticity is derived from the strain- and stress-optics laws (Neumann 1841, Maxwell 1852) on the theory of artificial double refraction (anisotropic birefringence) in a stressed isotropic, transparent solid. In 1816, Brewster coined the term photoelasticity due to the color pattern produced in clear glass when stressed and examined under polarized light. When certain transparent materials undergo stress, the material exhibits birefringence. As polarized light passes through the material, the rays refract and separate into two perpendicular components each parallel to the principal refractive indices of the material. A condition of the stress-optics laws states that these principal indices correspond to the principal stress directions [16].

In 2D cases, small-scale models with plane stress conditions are placed in a polariscope, which allows analysis of a model under polarized light. There are two types of optical interference patterns, *isoclinics* and *isochromatics*. Isoclinics designate the locus of all points where the principal stress directions are parallel to the directions of the polarizing axes, appearing as black bands. Isochromatics define the locus of all points having equal difference between the two principal stresses (constant maximum shear stress) appearing either as a field of dark fringes or a continuous range of the visible spectrum, depending on the light source (Figure 2) [17].

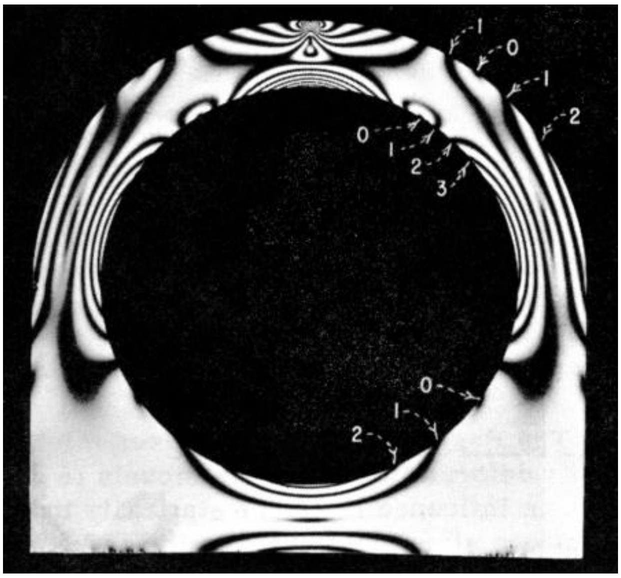


Fig. 2. Example Photoelasticity Pattern (outlet conduit with a concentrated load at the top and uniformly supported at base) [18]

Isoclinics allow the user to determine the principal stress directions at all points of the model, whereas the isochromatics indicate the differences in the principal stresses in the model and the stress at the free boundaries. As the isoclinics indicate the lines along which the directions of the principal stresses within the stressed model are constant, the isoclinics are not identical to the isostatics. Although isostatics can be drawn from the isoclinic diagram, the manual process is time-consuming to generate a full field of orthogonal isostatics [19].

While plane stress is an assumption of thin plate theory, thin plates subjected to transverse bending cannot be observed solely using the aforementioned 2D photoelasticity method. As thin plates subjected to transverse loading experience tension on one side and compression of equal absolute value on the opposite side, any phase difference resulting from light passing through the first half of the plate thickness would be cancelled out when travelling through the second half of the plate thickness [20], thus producing no photoelastic patterns. The first successful photoelasticity experiment to capture the principal stresses in plates subjected to transverse loading was documented by Goodier and Lee in 1941 [21], yet photographs of the optical interference patterns were not published. Based on the model discussed by Goodier and Lee, Kuske developed a "time-efficient" method using traditional 2D photoelasticity methods to view isoclinics occurring at the surface of the plate [20]. However, this method was not published until 1953, the same year as the completion of the Gatti Wool Factory and four years after the submittal of the 1949 isostatic ribbed floor patent.

Although Nervi referred to the photoelastic phenomenon as "the beauty and poetry of this transmutation of stress into a play of light rays [13]," photoelasticity experiments could not have been used to determine the ribbed floor slab patterns. Nervi asserted that photoelasticity was "more efficient in the study of local stresses in bodies of limited dimensions (crane hooks, chain links, and parts of machines), than in the analysis of entire structures [12]." Given this mindset, the authors assume that he would not have been inclined to use photoelasticity to evaluate the stresses present in a large flat slab. Despite his dedication to experimental methods, Nervi was cognizant of their time-consuming and costly aspects – the preparation, measurement, and analysis phases – and thus recommended that experimental modeling be reserved for structures of "special technical and architectural importance" and the analysis of structural problems unable to be solved theoretically [12]. The problem of determining the stresses in a thin plate subjected to transverse loading could be solved theoretically at the time of the patent submittal in 1949, indicating that the use of models to evaluate the stresses was not mandatory. This conclusion is substantiated by Iori [14] who asserted that the shape of the isostatics relating to the ribs of the Gatti Wool Factory must have been determined using calculations derived from thin plate theory.

2.3. **Mathematical Theory**

As no efficient experimental approach existed at the time, Arcangeli theoretically studied the concept of placing ribs along the isostatics of principal moments in proposing the idea to Nervi [14]. The two most commonly used plate theories are the Kirchhoff-Love and Reisner-Mindlin plate theories. The Kirchhoff-Love theory, applicable to thin plates, was developed by Love in 1888 using Kirchhoff's 1850 boundary condition assumptions [22]. Reissner-Mindlin plate theory, an extension of Kirchoff-Love plate theory and applicable to thick plates, takes into account shear deformations through the thickness of a plate and was proposed by Reissner in 1945, but not fully developed by Mindlin until 1951. Given this timeline, Arcangeli's theoretical calculations for the principal bending moment directions must have been based on Kirchoff-Love thin plate theory. Although thin plate theory involves high-order partial differential equations, numerous analytical (Navier, Lévy, Timoshenko [23]), approximate (Ritz), and design solutions (Westergaard and Slater) for thin plate-bending theory were already well-established and in widespread use when the patent for isostatic rib floors was filed in 1949. Additional resources were developed in Italy, including the analytical solutions of Botasso [22] and the design solutions of Santarella, who wrote and edited a plethora of practical manuals and theoretical texts on reinforced concrete produced as a result of the 1927 updates to the building regulations [3].

3. ISOSTATIC LINE TOOL

While Arcangeli and Nervi relied on theoretical calculations to determine the isostatics of principal moments for flat slabs, contemporary engineers can employ two manual methods. The first method includes theoretically calculating the principal bending moment directions at a selection of nodes, hand drawing lines at set lengths in the respective directions, recalculating the directions at the next nodes, and repeating the process until reaching a boundary [16]. This process is described in detail below and illustrated in Figure 3:

1) Select a start node (e.g., Node 0)
2) Calculate the maximum principal bending moment direction at that node
3) Draw a straight line of a set length in the calculated maximum principal bending moment direction (e.g., Segment a)
4) Recalculate the principal bending moment direction at the new node (e.g., Node 1)
5) Iterate through this method until crossing a boundary
6) Perform this method for a selection of start nodes to obtain a field of primary isostatics

To draw the secondary isostatics, one can simply use the minimum principal bending moments.

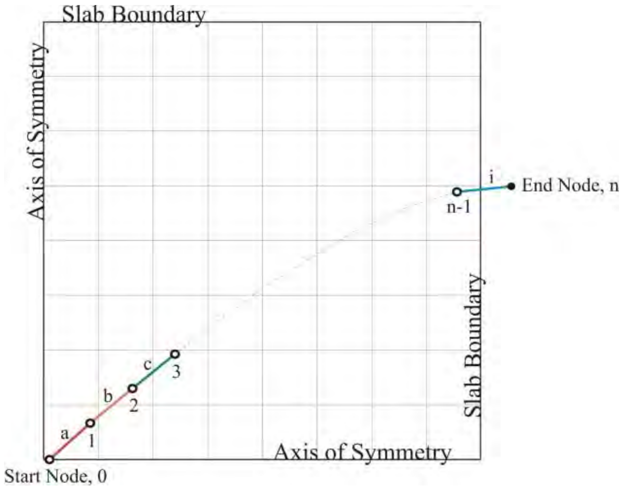


Fig. 3. Isostatic Path Determination

Alternatively, one can print out the principal bending moment trajectories displayed by commercially available Finite Element Software and manually draw the isostatics tangent to these trajectories or import a picture of the plot into a CAD or graphics editing program to draw the lines electronically. The main disadvantages of these methods are: 1) the plots produced by commercial FEM software are typically of low resolution, which complicates drawing lines at the accurate angles; 2) the principal bending moment directions are averaged to the center of the finite element, which leads to biased isostatics; and 3) the principal bending moment trajectories are displayed as arrows, rather than continuous lines, which forces the drawer to interpolate between elements.

As an alternative to these manual drawing options, an automatic, computational method of drawing the isostatics was developed using the FE Software SAP 2000v15 and MATLAB. First, a flat slab FE model is generated with the appropriate geometry, external loading, boundary conditions, and material properties. After running the analysis, the primary and secondary principal bending moment angles are obtained. The MATLAB algorithm follows the six-step procedure previously outlined. Termination of the algorithm occurs when the generated isostatic line has reached a slab boundary, resulting in a plot of the isostatic line (Figure 3).

4. ANALYSES OF THREE FLOOR SLABS

Three of Nervi's floor slabs are evaluated using the Isostatic Line Tool: 1) the Gatti Wool Factory warehouse floor (1953, Rome), 2) the Palace of Labor mezzanine floor (1961, Turin), and 3) the Large Sports Palace gallery floor (1960, Rome). As all of these floors were designed to support uniformly-distributed loading conditions, and as scaling the load does not alter the directions of the principal bending moments, each system is evaluated under equivalent uniformly distributed loads. This consistency in the applied loading patterns leaves the principal bending moments unbiased so as to highlight how alterations of the dimensions and boundary conditions influence the resulting isostatic rib patterns.

4.1. **Gatti Wool Factory**

The use of isostatics to inform the arrangement pattern for a ribbed floor system was first realized for the warehouse floor of the Gatti Wool Factory. Nervi and Arcangeli collaborated with architect Carlo Cestelli Guidi for the factory design, which required a wide-spanning floor system capable of supporting heavy wool-spinning machinery [14]. The curved isostatic pattern of the Gatti Wool Factory floors (Figure 4) fully exploited the moldable flexibility of Nervi's ferrocement forms.

Fig. 4. Gatti Wool Factory Floor System [13]

Each 5m x 5m slab of the Gatti Wool Factory is supported by a central column. All slabs are monolithically joined along the perimeter edges [14]. A quarter of the slab is evaluated using the Isostatic Line Tool. To satisfy the one-eighth symmetry in each quarter slab, a sufficiently fine step size must be used. A relation between element length and step size was prescribed to evaluate symmetry convergence, as illustrated in Figure 5, with the chosen step size of one-sixty-fourth the element length yielding a symmetry error of only 0.31%.

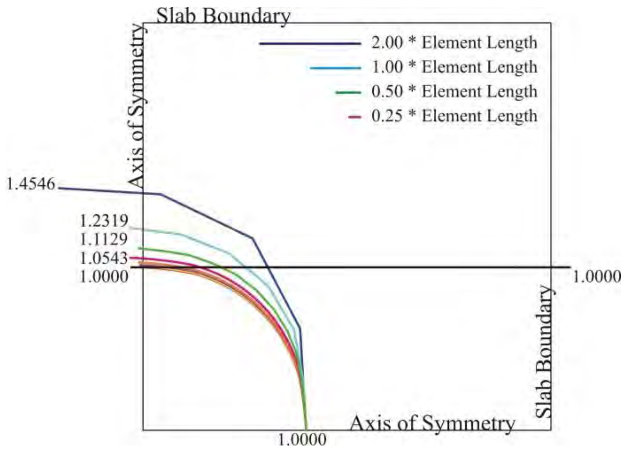


Fig. 5. Stepping Procedure Symmetry Convergence

The analysis of an upper-right quarter of a 5m x 5m Gatti Wool Factory slab, using the Isostatic Line Tool, is shown in Figure 6. Symmetry conditions (axial rotation and orthogonal translation restrained) are applied to the left and bottom boundaries and monolithic perimeter conditions (full rotational fixity and in-plane translation restraints) are modeled on the top and right boundaries.

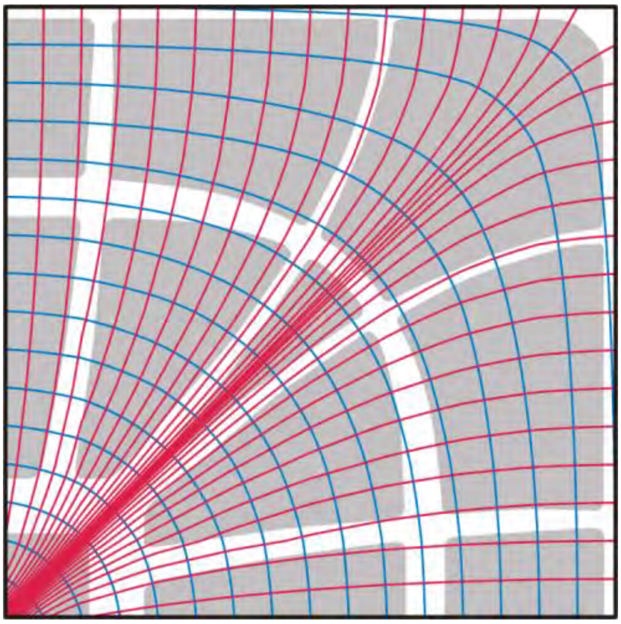


Fig. 6. Gatti Wool Factory Quarter Slab Analysis

The red lines in Figure 6 represent the primary isostatics, corresponding to the maximum principal bending moments, and the blue lines represent the secondary isostatics, corresponding to the minimum principal bending moments. Both pairs of lines were generated from start nodes set at increments equal to half of the finite element lengths and placed along all four boundaries. The concentrations of lines indicate convergence of the principal bending moment directions, highlighted by the prescribed method of starting node generation. The grey and white outline shows an approximate plan of the Gatti Wool Factory floor to illustrate the correlation between the theoretical isostatics and the as-built rib pattern. The concentric secondary isostatics produced around the column support at the bottom left of the quarter slab reflects the reasoning for placing concentric curves of reinforcement around the column head and verifies Nervi's placement of the rib encircling the column head. The ribs emanating from the central column also show close correlation to the generated primary isostatics.

Fig. 7. Palace of Labor Floor System [13]

4.2. **Palace of Labor (Palazzo del Lavoro)**

In 1961, a celebration was held in Turin for the centennial of Italy's unification, for which Nervi designed and constructed the Palace of Labor [24]. The interior border along the building perimeter is formed by the mezzanine, which has repeated reinforced concrete slabs with isostatic ribs (Figure 7). Each 10m x 10m slab is supported by columns at the four corners and the isostatic patterns follow one-eighth symmetry [13]. The analysis of an upper-right quarter of a 10m x 10m Palace of Labor slab is shown in Figure 8, with symmetry conditions at the left and bottom boundaries, monolithic perimeter conditions at the top and right boundaries, and a column support at the top right corner. The presented results show strong correlation exists between the theoretical primary and secondary isostatics and the as-built plan.

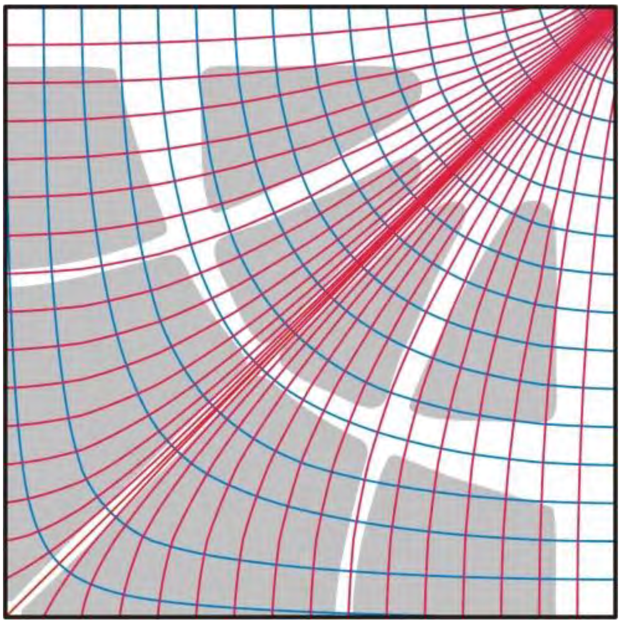
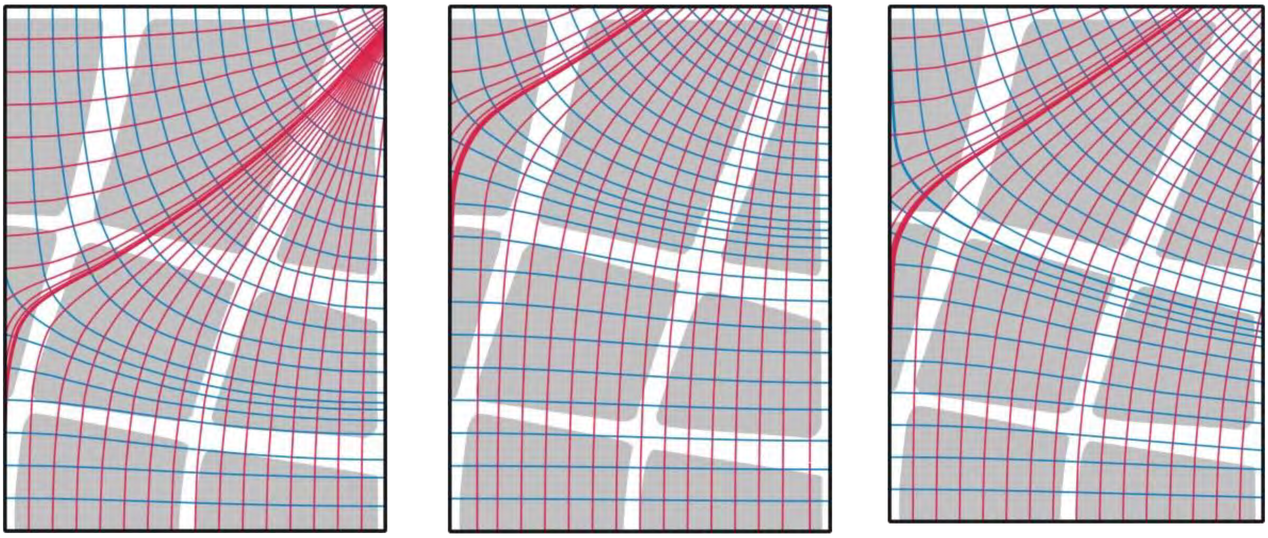


Fig. 8. Palace of Labor Quarter Slab Analysis

4.3. **Large Sports Palace (Palazzo dello Sport)**

The Large Sports Palace was designed by Nervi, with architect Marcello Piacentini, as one of three structures constructed for the 1960 Rome Summer Olympic Games. The intradoses of the perimeter gallery floor include isostatic rib patterns (Figure 9) [25]. In contrast to the Gatti Wool Factory and Palace of Labor slabs, the Large Sports Palace slabs are rectangular. While these slabs are supported by columns at the four corners, the shift from a square to rectangular boundary shape induces different isostatic patterns.

Fig. 9. Large Sports Palace Floor System [25]



Figs. 10-12. Large Sports Palace Quarter Slab Analysis – Full Fixity, Partial Fixity, Simply-Supported

The analysis of the Large Sports Palace floor required further evaluation as applying identical boundary conditions as the Palace of Labor case study yielded isostatics satisfying the assumed bounds of only two of the four outlined floor ribs (Figure 10). Additional boundary conditions were evaluated to try to shift the isostatics towards the two disparate ribs, yet no combination yielded exact results for all ribs, as seen in the Gatti Wool Factory and Palace of Labor analyses. The analysis shown in Figure 11 removed the fixity of the top boundary and the Figure 12 analysis placed simply-supported conditions at the top and right boundaries. While the fully simply-supported conditions show close correlation between the topmost transverse rib and the neighboring secondary isostatics, the other three ribs lack correlation with the isostatics. The rib closest to the left boundary captures the strictly vertical portions of select primary isostatics, whose starting nodes start along the left axis of symmetry, yet they soon diverge towards the top-right corner support.

Despite this divergence, the two disparate ribs from Figure 10 appear to be offsets of the two curves closely correlated to the isostatics. This departure from pure conformity to the isostatics supports Nervi's statement of prescribing the design to an interpretation of the true static behavior. Naturally, the addition of the ribs dictates the structural behavior of the floor system, reorienting the isostatics along these predefined paths.

5. CONCLUSION

The successes of Nervi's isostatic floor systems and the discussed historical precedents highlight the catalysts for the widespread recognition of a reinforced concrete floor system: 1) the protection of patents; 2) the personal ownership or financial sponsorship of an established engineering firm; and 3) the connection to academic research and experimentation. In addition to these influential factors, the artisanal construction sites endemic to Italy and the political atmosphere at the time helped foster the construction and material innovations developed by Nervi related to floor system design. The elegant design of these floor systems is expressed by the rib patterns, which allude to the isostatics of the principal moments produced in a corresponding flat slab system. The experimental stress analysis methods (strain gauge and photoelasticity) referenced by Nervi and the mathematical analysis methods available at the time (pertaining to transversely loaded flat slabs) were discussed to evaluate the potential for these methods to have been used by Nervi to generate the isostatics. Although isostatics can be generated via these experimental and theoretical methods, it was determined that a theoretical approach was used by Nervi and Arcangeli.

The theoretical approach to generating isostatics has been implemented in a computational Isostatic Line Tool, the efficacy of which was proven in analyses of three of Nervi's floor systems. A modern designer can explore the efficiency of the Isostatic Line Tool to develop conceptual designs for ribbed floors according to new geometries, boundary conditions, and loading patterns. This tool provides designers with a visual method of automatically determining the primary and secondary isostatics in flat slabs, which provides an efficient alternative to the potentially biased, time-consuming FE-based drawing methods composed of centrally averaged principal bending moment trajectories. The close correlation between the theoretical isostatics and the as-built rib patterns for the evaluated floor slabs, as well as the artistic interpretation of the static theory, highlights Nervi's desire to construct correctly while expressing elegance.

6. ACKNOWLEDGEMENTS

The authors are grateful for the financial support of the Sherrerd Fund and Princeton University. The authors would like to thank Marco Nervi (Pier Luigi Nervi Project Association) for permitting the use of the images of Nervi's floor systems. The authors would also like to thank Serguei Bagrianski (Princeton University) for his help in developing the Isostatic Line Tool and Ted Segal (Princeton University) for his research recommendations.

7. REFERENCES

[1] Friedman, D., *Historical Building Construction: Design, Materials, and Technology*, 2nd Ed, W.W. Norton & Co (2010).

[2] Addis, B., *Building: 3000 Years of Design, Engineering and Construction*, Phaidon Press (2007).

[3] Iori, T., *150 anni di storia del cement in Italia: Le opera, gli uomini, le imprese*, Gangemi Editore (2011).

[4] Gulli, R., *Struttura e costruzione – Structure and construction*, Firenze University Press (2008).

[5] Poretti, S., *Le plancher en poteries et béton dans le modernisme italien,* In: Gargiani, R. (ed) *L'architrave le plancher la plate-forme: Nouvelle histoire de la construction*, Presses polytechniques et universitaires romandes, Lausanne (2012).

[6] Sorrentino, L., *The Early Entrance of Dynamics in Earthquake Engineering: Arturo Danusso's Contribution*, ISET J. Earthquake Tech., Vol. 44, No. 1 (2007) pp. 1-24.

[7] Levi, F., and Chiorino, M.A., *Concrete in Italy. A review of a century of concrete progress in Italy, Part 1: Technique and architecture*, Concrete Int,, Vol. 26, No. 9 (2004) pp. 55-61.

[8] Greco, C., *Muggia e Nervi, Il Fecondo e Complesso Rapporto tra un Grande Maestro e un Discepolo Illustre*, In: Bettazzi, M.B. and Lipparini, P. (ed) *Attilio Muggia : una storia per gli ingegneri*, Editrice Compositori, Bologna (2010).

[9] Pourtois, C., *Biography*, Olmo, C., and Chiorino, C. (eds), *Pier Luigi Nervi: Architecture as Challenge*, Silvana Editoriale (2010) pp. 201-209.

[10] Iori, T., and Poretti, S., *Pier Luigi Nervi's Works for the 1960 Rome Olympics*, Actas del Cuarto Congreso Nacional de Historia de la Construccion (2005).

[11] Chiorino, C., *Eminent Structural Engineer: Pier Luigi Nervi (1891-199) – Art and Science of Building*, Struct Eng Int, No. 1 (2010) pp. 107-109.

[12] Nervi, P.L., *Structures*, Salvadori, G., and Salvadori, M. (trans), F.W. Dodge Corp. (1956).

[13] Nervi, P.L., *Aesthetics and Technology in Building*, Harvard University Press (1965).

[14] Iori, T., *Le plancher a nervures isostatiques de Nervi*, In: Gargiani, R. (ed) *L'architrave le plancher la plate-forme: Nouvelle histoire de la construction*, Presses polytechniques et universitaires romandes, Lausanne (2012).

[15] Dally, J.W., and Riley, W.F., *Experimental Stress Analysis*, 3rd Ed., McGraw-Hill (1991).

[16] Jessop, H.T., and Harris, F.C. *Photoelasticity: Principles & Methods*, Dover Publications, Inc. (1950).

[17] Broniewska, C., and Mitra, M.S., *Photoelastic Method for Stress Analysis*, Symposium on Industrial Failure of Engineering Metals & Alloys (1953).

[18] Moody, W.T., and Phillips, H.B., *Photoelastic and Experimental Analog Procedures*, US Dept. Int. Bureau of Reclamation, No. 23 (1967) pp. 1-100.

[19] Heywood, R.B., *Photoelasticity for designers*, Pergamon Press (1969).

[20] Kuske, A., *Photo-elastic Research on Plates in Transverse Bending*, Engineering (1953), pp. 20-39.

[21] Goodier, J.N., and Lee, G.H., *An Extension of the Photoelastic Method of Stress*, Trans. Amer. Soc. Mech. Eng., Vol. 63 (1941) pp. A-27–A-29.

[22] Westergaard, H.M., and Slater, W.A., *Moments and Stresses in Slabs*, Proc. ACI, Vol. 17 (1921) pp.1-124.

[23] Timoshenko, S., *Theory of Plates and Shells*, McGraw-Hill (1959).

[24] Chiorino, C., *The Palazzo del Lavoro,* Olmo, C., and Chiorino, C. (eds), *Pier Luigi Nervi: Architecture as Challenge*, Silvana Editoriale (2010) pp. 168-173.

[25] Nervi, P.L., *New Structures*, Architectural Press (1963).